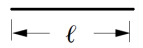

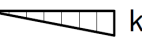
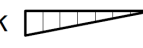
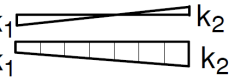
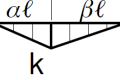
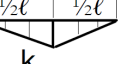
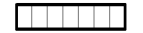


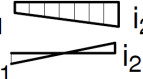








# Formänderungsintegrale

$$\delta_{ik} = \int_0^{\ell} f_i(x) f_k(x) dx$$

Nr					 Anmerkung 1 beachten!	 Anmerkung 2 beachten!	
0	$\int k^2 dx$	$k^2 \ell$	$\frac{1}{3} k^2 \ell$	$\frac{1}{3} k^2 \ell$	$\frac{1}{3} (k_1^2 + k_1 k_2 + k_2^2) \ell$	$\frac{1}{3} k^2 \ell$	$\frac{1}{3} k^2 \ell$
1		$ik \ell$	$\frac{1}{2} ik \ell$	$\frac{1}{2} ik \ell$	$\frac{1}{2} i (k_1 + k_2) \ell$	$\frac{1}{2} ik \ell$	$\frac{1}{2} ik \ell$
2		$\frac{1}{2} ik \ell$	$\frac{1}{3} ik \ell$	$\frac{1}{6} ik \ell$	$\frac{1}{6} i (k_1 + 2k_2) \ell$	$\frac{1}{6} ik (1 + \alpha) \ell$	$\frac{1}{4} ik \ell$
3		$\frac{1}{2} ik \ell$	$\frac{1}{6} ik \ell$	$\frac{1}{3} ik \ell$	$\frac{1}{6} i (2k_1 + k_2) \ell$	$\frac{1}{6} ik (1 + \beta) \ell$	$\frac{1}{4} ik \ell$
4		$\frac{1}{2} k (i_1 + i_2) \ell$	$\frac{1}{6} k (i_1 + 2i_2) \ell$	$\frac{1}{6} k (2i_1 + i_2) \ell$	$[\frac{1}{6} i_1 (2k_1 + k_2) + \frac{1}{6} i_2 (k_1 + 2k_2)] \ell$	$k [\frac{1}{6} i_1 (1 + \beta) + \frac{1}{6} i_2 (1 + \alpha)] \ell$	$\frac{1}{4} k (i_1 + i_2) \ell$
5	 Anmerkung 2 beachten!	$\frac{1}{2} ik \ell$	$\frac{1}{6} ik (1 + \gamma) \ell$	$\frac{1}{6} ik (1 + \delta) \ell$	$\frac{1}{6} i [k_1 (1 + \delta) + k_2 (1 + \gamma)] \ell$	$\gamma > \alpha: \frac{ik}{6} \left[ 2 - \frac{(\gamma - \alpha)^2}{\gamma(1 - \alpha)} \right] \ell$ $\gamma < \alpha: \frac{ik}{6} \left[ 2 - \frac{(\alpha - \gamma)^2}{\alpha(1 - \gamma)} \right] \ell$ $\gamma = \alpha: \frac{1}{3} ik \ell$	$\gamma > 1/2: \frac{ik}{6} \left[ 2 - \frac{(\gamma - 0.5)^2}{0.5 \gamma} \right] \ell$ $\gamma < 1/2: \frac{ik}{6} \left[ 2 - \frac{(0.5 - \gamma)^2}{0.5(1 - \gamma)} \right] \ell$ $\gamma = 1/2: \frac{1}{3} ik \ell$
6	 Quadr. Parabel	$\frac{2}{3} ik \ell$	$\frac{1}{3} ik \ell$	$\frac{1}{3} ik \ell$	$\frac{1}{3} i (k_1 + k_2) \ell$	$\frac{1}{3} ik (1 + \alpha \beta) \ell$	$\frac{5}{12} ik \ell$
7	 Quadr. Parabel	$\frac{2}{3} ik \ell$	$\frac{1}{4} ik \ell$	$\frac{5}{12} ik \ell$	$\frac{1}{12} i (5k_1 + 3k_2) \ell$	$\frac{1}{12} ik (5 - \alpha - \alpha^2) \ell$	$\frac{17}{48} ik \ell$
8	 Quadr. Parabel	$\frac{2}{3} ik \ell$	$\frac{5}{12} ik \ell$	$\frac{1}{4} ik \ell$	$\frac{1}{12} i (3k_1 + 5k_2) \ell$	$\frac{1}{12} ik (5 - \beta - \beta^2) \ell$	$\frac{17}{48} ik \ell$
9	 Quadr. Parabel	$\frac{1}{3} ik \ell$	$\frac{1}{4} ik \ell$	$\frac{1}{12} ik \ell$	$\frac{1}{12} i (k_1 + 3k_2) \ell$	$\frac{1}{12} ik (1 + \alpha + \alpha^2) \ell$	$\frac{7}{48} ik \ell$
10	 Quadr. Parabel	$\frac{1}{3} ik \ell$	$\frac{1}{12} ik \ell$	$\frac{1}{4} ik \ell$	$\frac{1}{12} i (3k_1 + k_2) \ell$	$\frac{1}{12} ik (1 + \beta + \beta^2) \ell$	$\frac{7}{48} ik \ell$

Anmerkung 1: Zeile 4 und Spalte 4 gelten auch für  $i_1 > i_2$  bzw.  $k_1 > k_2$ , sowie für negative  $i$ - bzw.  $k$ -Werte (Werte immer mit Vorzeichen einsetzen!).

Anmerkung 2: Zeile 5 und Spalte 6:  $\alpha, \beta, \gamma, \delta$  sind Bruchteile der Abschnittslänge  $\ell$ , d.h. zwischen 0 und 1; z.B.: Dreiecksspitze in der Mitte:  $\gamma = \delta = 0.5$

Anmerkung 3: Zeilen 6 bis 10: Der Punkt • markiert den Scheitel der quadratischen Parabel.